Covering Arrays, Set Covers, Algorithms and their Complexity

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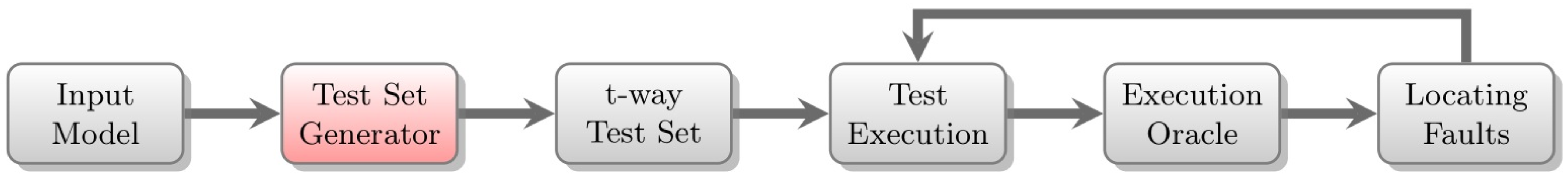
Generation of Covering Arrays for Abstract Combinatorial Test Suites

Covering Arrays for Combinatorial Testing

- Covering Arrays (CAs) provide the theoretical means for Combinatorial Testing (CT)
- Columns of a CA map to the parameters of a system under test.
- Rows of a CA encode the individual test cases.
- Their combinatorial properties guarantee that derived test sets **cover** all **t-way interactions**.
- To apply CT to arbitrary SUTs, we need to be able to generate arbitrary CAs.



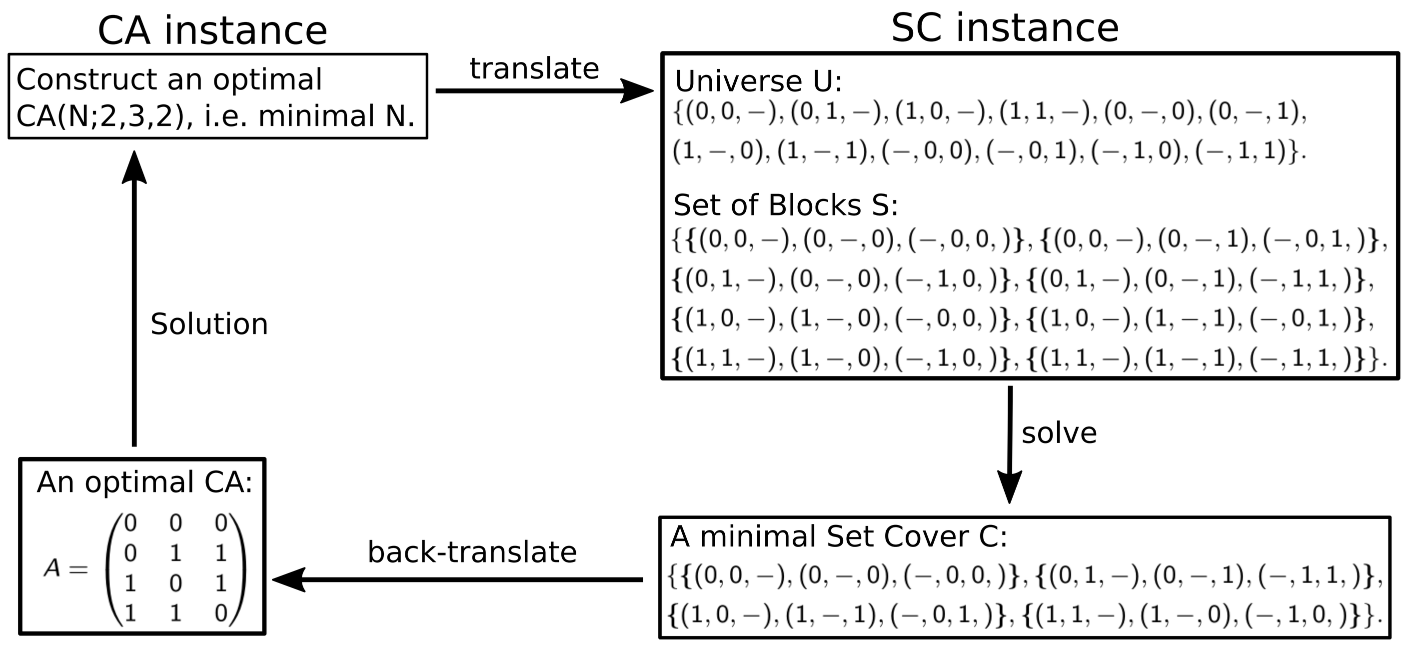
The Covering Array Generation Problem

- Given a **strength t**, a number of columns k and the respective columns' alphabet sizes $v_1, \dots, v_k$.
- Construct a (mixed) covering array $\mathsf{MCA}(N; t, k, (v_1, \dots, v_k))$ minimizing the number of rows N.
- Exact and direct constructions of CAs exist only for some corner cases.
- **For general applications we need heuristic algorithms for arbitrary CA generation.**

Covering Arrays via Set Covers

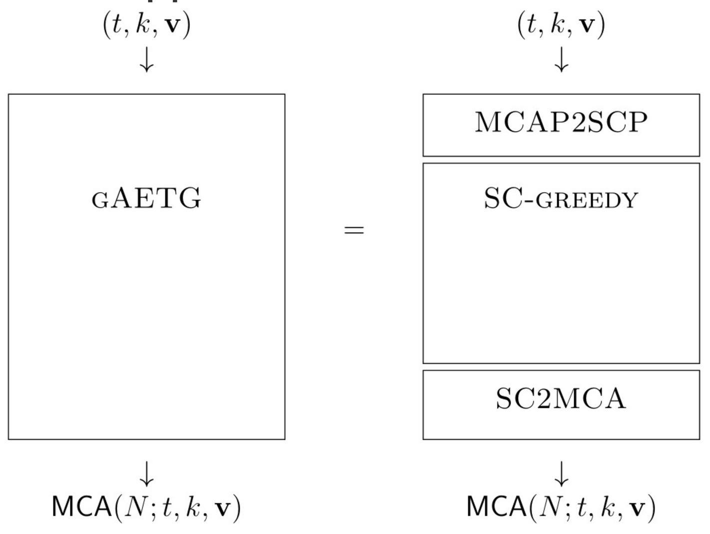
Optimal Covering Arrays as Minimal Set Covers

- The Set Cover Problem is a well studied problem in theoretical CS.
- For a given universe U and a set of blocks $\mathcal{S}$, i.e. subsets of U, we want to find a minimal subset of $\mathcal{S}$ that covers U.
- The CA generation problem can be interpreted as a Set Cover problem:
  - $U := \mathbb{T}_t$ the set of all t-way interactions
  - $\mathcal{S} := \prod_{i=1}^{k} [v_i]$ set of potential rows
  - Then a **minimal set cover** represents an **optimal CA**



Algorithms for Covering Arrays via Set Covers

- This connection allows to apply Set Cover (SC) Algorithms for CA generation.
- Some existing algorithms for CA generation can be identified as classical SC algorithms applied to CA instances.



- Allowing to import approximations and bounds for CAs:

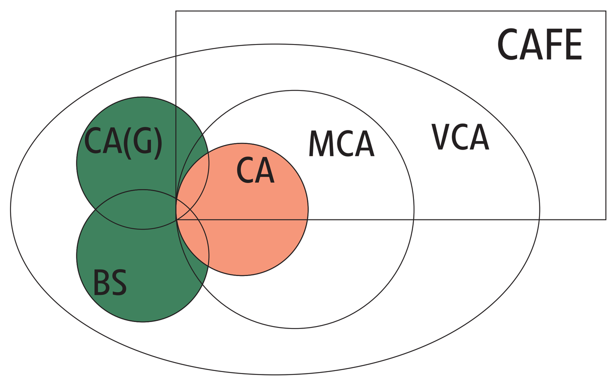
$$N \leq \mathit{MCAN}(N; t, k, \mathrm{v}) \cdot \log \binom{k}{t}$$

- This connection can be generalized to **weighted budgeted** instances pertaining to **weighted budgeted CA construction.**

Covering Arrays and Computational Complexity

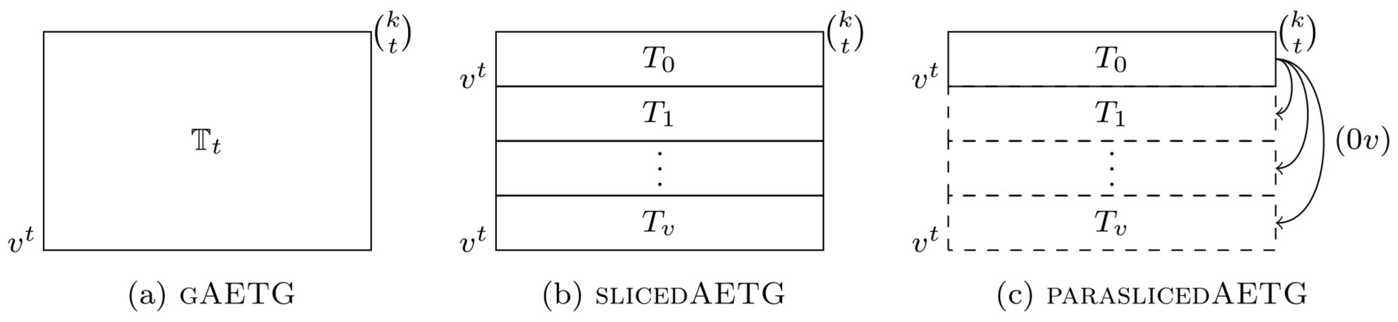
- Formulation of CA-related problems as formal complexity problems.
- Establish connections between these problems:
  - For arbitrary but fixed t and v, it holds that
    (i) $\mathit{decSizeOMCA}_{t,v} \leq_p^T \mathit{detSizeOMCA}_{t,v} \leq_p^T \mathit{genOMCA}_{t,v}$.
    (ii) $\mathit{decSizeOMCA}_{t,v} \equiv_p^T \mathit{detSizeOMCA}_{t,v}$.
- Analyse state of the art of complexity problems related to CAs.
- Correction of statements and clarification of misinterpretation.
- The computational complexity of the Covering Array generation problem remains unknown.

Classes of Covering Arrays	Decide Existence	Decide Size	Determine Size	Generation
optimal $CA_{2,2}$	P	P	P	P
optimal $CA_{t,v}$	P	NP	???	???
optimal $MCA_{t,v}$	P	NP	???	???
optimal BS_d	P	NP-complete	NP-hard	NP-hard
optimal $VCA_{\tau,2}$	P	NP-complete	NP-hard	NP-hard
optimal $VCA_{\tau,v}$	P	NP	???	???
optimal VCA_{τ}	P	NP-hard	NP-hard	NP-hard
optimal CA(G)	P	NP-complete	NP-hard	NP-hard
CAFE	NP-complete	NP-hard	NP-hard	NP-hard



slicedAETG: A specialized Algorithm for CA construction

- The CA generation problem has more inherent structure compared to the general Set Cover problem.
- This can be exploited in order to devise more efficient algorithms.
- The slicedAETG algorithm is a specialization of a general greedy algorithm that is tailored to suite the CA generation problem.



Schematics how different algorithms process the set of all t-way interactions $\mathbb{T}_t$.

	$\geq$ # Rows	Runtime	Memory for T
gAETG	$v^t \ln(v^t \binom{k}{t}) + 1$	$O\left(v^{k+t} t \binom{k}{t} \ln(v^t \binom{k}{t})\right)$	$\Theta(v^t \binom{k}{t})$
slicedAETG	$v^{t+1} \ln(v^{t-1} \binom{k}{t}) + v$	$O\left(v^{k+t} t \binom{k}{t} \ln(v^{t-1} \binom{k}{t})\right)$	$\Theta(v^{t-1} \binom{k}{t})$
paraslicedAETG	$v^{t+1} \ln(v^{t-1} \binom{k}{t}) + v$	$O\left(v^{k+t-1} t \binom{k}{t} \ln(v^{t-1} \binom{k}{t})\right)$	$\Theta(v^{t-1} \binom{k}{t})$

Bounds on number of rows of output CAs, runtime and memory usage.

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