Neural Networks for Covering Arrays

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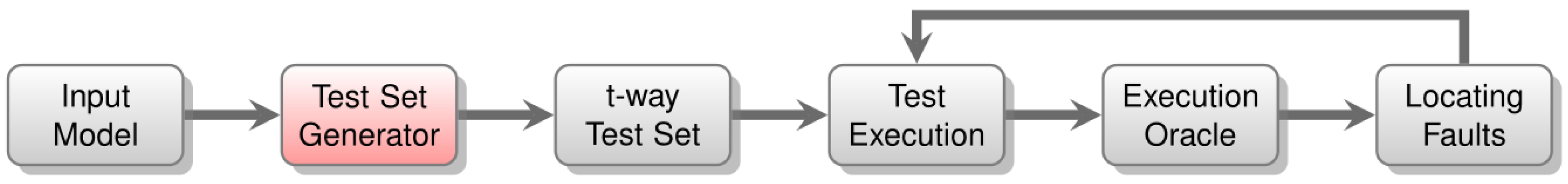
Covering Array Optimization

Covering Arrays

- Covering Arrays (CAs) are combinatorial structures used in Combinatorial Testing.
- They guarantee that every t-way interaction appears in at least one row (test).
- A uniform Covering Array is denoted as CA($N;t,k,v$), where N is the number of rows, t is the strength, k is the number of columns and v is the cardinality of the alphabet.
- CAs with the smallest number of rows possible are called optimal CAs.



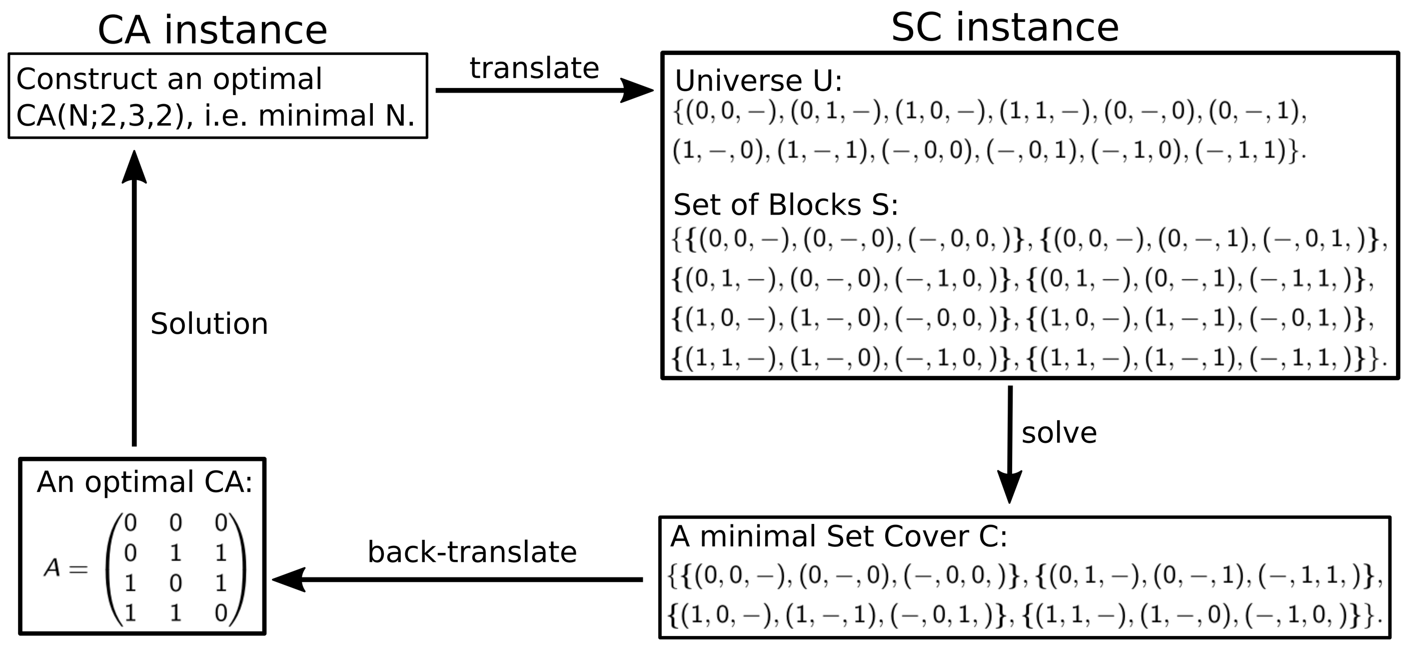
The Covering Array Generation Problem

- Generating optimal CAs is tightly coupled to hard combinatorial optimization problems.
- Commonly used generation methods include greedy algorithms, mathematical constructions and metaheuristic approaches.
- We investigated the use of neural networks for covering array generation.

Boltzmann Machines for CA Generation

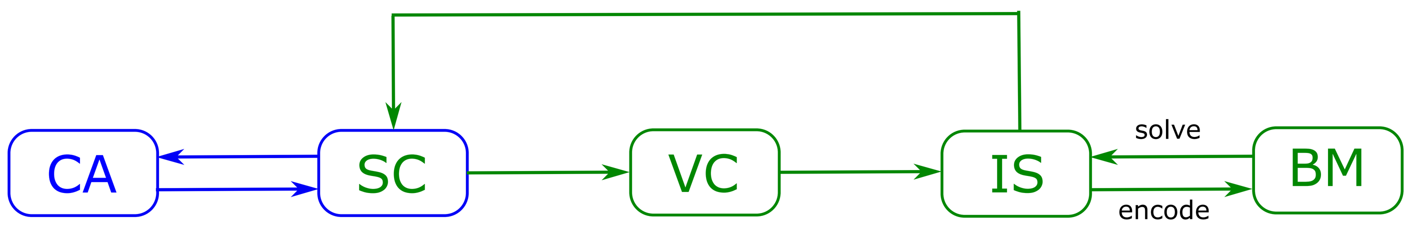
Covering Arrays as Set Covers:

- For a given universe U and a set of blocks $\mathcal{S}$, i.e. subsets of U, we want to find a minimal subset of $\mathcal{S}$ that covers U.
- The CA generation problem can be interpreted as Set Cover problem:
  - $U := \mathbb{T}_t$ the set of all t-way interactions
  - $\mathcal{S} := \prod_{i=1}^{k}[v_i]$ set of potential rows
  - Then a *minimal set cover* represents an *optimal CA*.



Boltzmann Machines:

- Underlying graph: vertices = neurons, edges = synapses.
- Consensus function: Sum of weights of all active edges.
- State changes are stochastic.
- Objective: Find optimal allowable state.



From CAs to Boltzmann Machines:

- CA generation problem encoded as a Minimal Set Cover problem.
- SC problem further encoded as Minimum Vertex Cover (VC) problem.
- The graph is underlying the Boltzmann machine.
- Simulated Annealing is used to optimize the state of the Boltzmann machine and solve the complement of the VC problem, the Maximal Independent Set (IS) problem.
- Convert the obtained IS back to a Covering Array.

Algorithm 1 BMforCA

INPUT: t, k
Require: ϵ
$G'_{t,k} \leftarrow$ InitialGraph($G_{t,k}$)
$\omega(G'_{t,k}) \leftarrow$ InitialWeight($G'_{t,k}, \epsilon$) ▷ Assign weights
$\mathcal{I} \leftarrow$ SA($G'_{t,k}, \omega(G'_{t,k})$)
return CA($|V| - |\mathcal{I}|; t, k, 2) = V \setminus \mathcal{I}$

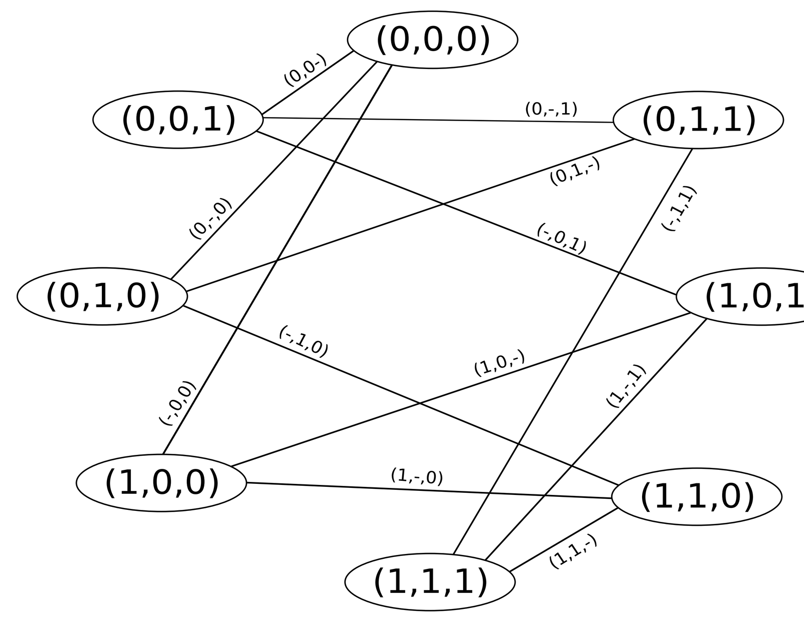


Figure 1: The instance CA(N; 2, 3, 2) encoded on a Graph

Constructing CAs using Hopfield NNs

- The CA instance is encoded on a graph as a SC instance.
- The goal is to minimize the number of activated neurons, in order to find a minimal covering array.
- This is encoded in the energy function of the Hopfield NN:

$$E = \alpha \sum_{i \in I} s_i + \beta \sum_{u \in U} \left(d - \sum_{i \in I} \chi(u, S_i) s_i \right)^2$$

- Optimized via a deterministic greedy SC algorithm.
- A binary search for the parameter d is used to optimize the energy function for generation of small CAs.

Contributions

- First time neural networks were successfully used for the construction of Covering Arrays.
- Thanks to updates of the weights of edges and the structure of the underlying graph, the system demonstrates learning behaviour:

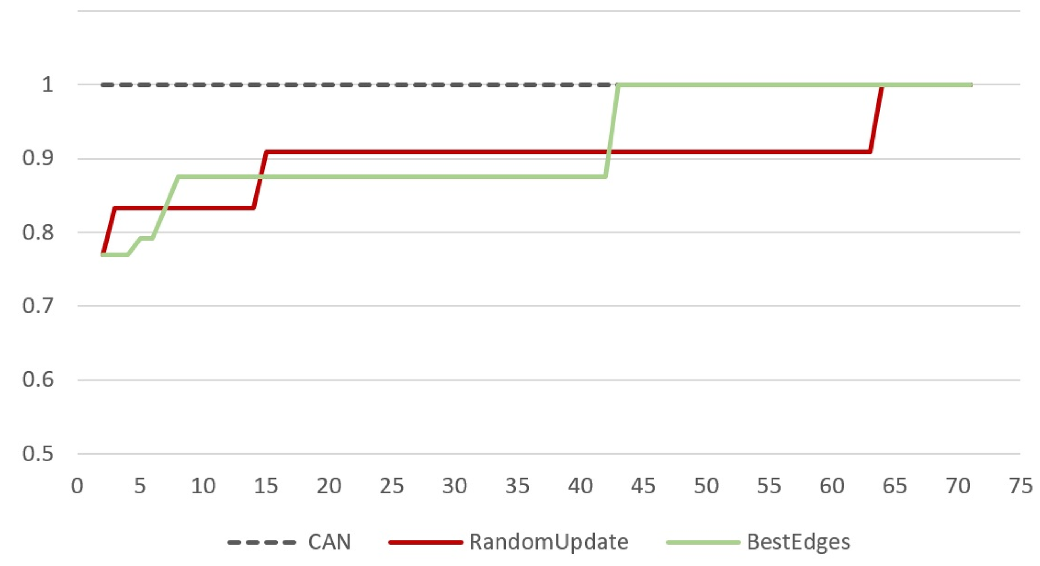


Figure 2: The BM neural network converges towards an optimal solution.

Ludwig Kampel, Manuel Leithner, Bernhard Garn, and Dimitris E. Simos. Problems and algorithms for covering arrays via set covers. *Theoretical Computer Science*, 800:90 – 106, 2019. Special issue on Refereed papers from the CAI 2017 conference.
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